# Best Arm Identification with Quantum Oracles

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## Classical Best Arm Identification

Find the best treatment. Find the most profitable ad.

- $K$ arms: each with a Bernoulli r. v. $\mathcal{B}(\mu_k)$ with unknown mean $\mu_k$
  - $\Delta_k = \mu^* - \mu_k$
- In each time slot $t$, pull one arm, and observe the arm reward realization.
- With confidence $1-\delta$, find the best arm $k^* \coloneqq \arg\max \mu_k$ with as small number of samples as possible (sample complexity)

Image credit: https://www.flaticon.com/

## Technique 1: Quantum Parallelism Permits More Information.

To Achieve $\mathbb{P}(|\hat{\mu}_k - \mu_k| \le \epsilon) \ge 1-\delta$,

- Classically: $O\left(\frac{1}{\epsilon^2}\log\frac{1}{\delta}\right)$ samples
- Quantumly: $O\left(\frac{1}{\epsilon}\log\frac{1}{\delta}\right)$ samples.

## Technique 2: Amplitude Amplification + Binary Search.

- Query all arms coherently
- Shrink the potential optimal arms efficiently

## Technique 3: Arm Set Partition.

- Partition $K$ arms to $\frac{K}{m}$ subsets
- Eliminate bad arms in the subset level

## Quantum Best Arm Identification

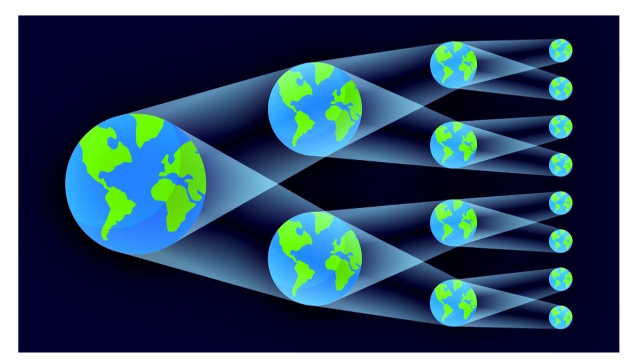
Find the best channel in the quantum network: $\mathcal{O}_1$, $\mathcal{O}_2$, $\mathcal{O}_3$

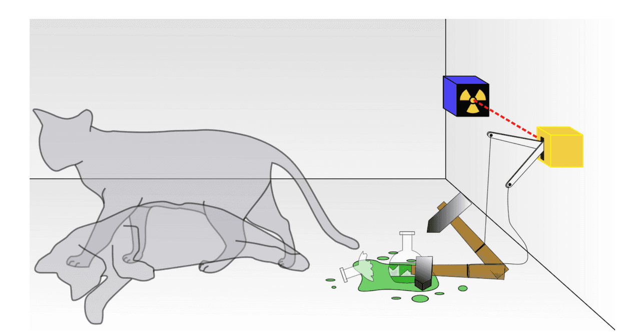
Find the best quantum solver

### Weak quantum oracle (Parallelism)

Measure the state: $\mathcal{B}(\mu_k) \sim \begin{cases} |1\rangle & \text{w. p. } \mu_k \\ |0\rangle & \text{w. p. } 1-\mu_k \end{cases}$

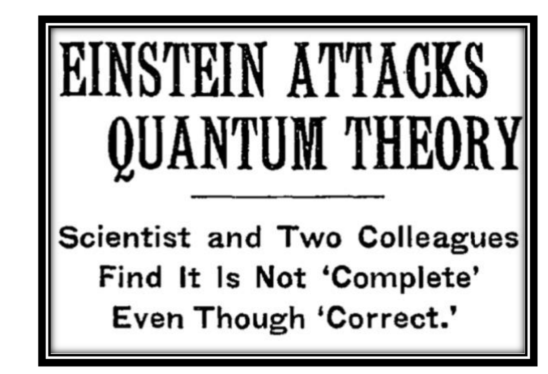
$$\mathcal{O}_k\colon\ |0\rangle \mapsto \sqrt{\mu_k}|1\rangle + \sqrt{1-\mu_k}|0\rangle$$



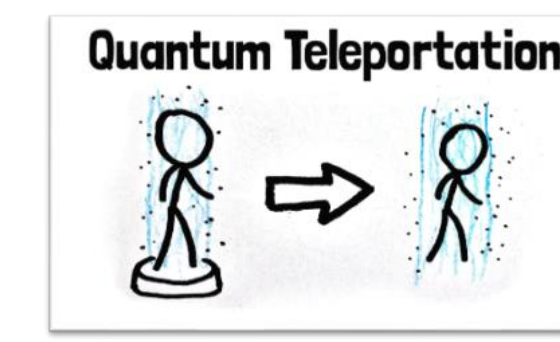


### Strong quantum oracle (Entanglement)

$$\mathcal{O}_{\text{MAB}}\colon \sum_{k=1}^{K} a_k|k\rangle_I|0\rangle_R \mapsto \sum_{k=1}^{K} a_k|k\rangle_I(\sqrt{1-\mu_k}|0\rangle_R + \sqrt{\mu_k}|1\rangle_R)$$


EINSTEIN ATTACKS QUANTUM THEORY

Scientist and Two Colleagues Find It Is Not 'Complete' Even Though 'Correct.'



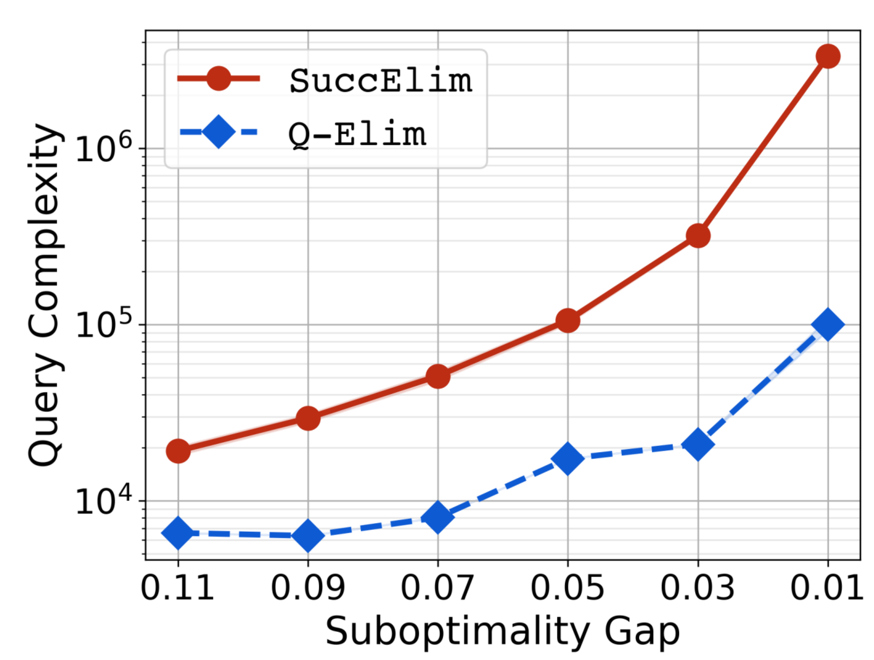

### $m$-Constrained Quantum Oracle (Unified)

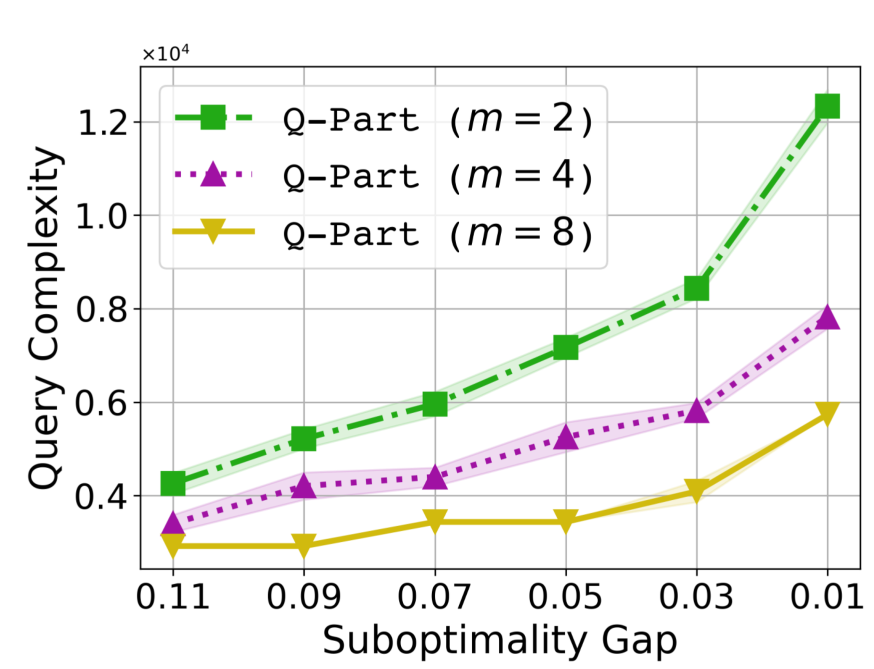
Weak ($m=1$) Strong ($m=K$)

General $m$-Constrained ($1<m<K$)

$$\mathcal{O}_{\mathcal{S}}\colon \sum_{k\in\mathcal{S}} a_k|k\rangle_I|0\rangle_R \mapsto \sum_{k\in\mathcal{S}} a_k|k\rangle_I(\sqrt{1-\mu_k}|0\rangle_R + \sqrt{\mu_k}|1\rangle_R)$$

## Simulations




| Oracle | Lower Bound | Upper Bound |
|---|---|---|
| Classical | $\Omega\left(\sum_k \frac{1}{\Delta_k^2}\log\frac{1}{\delta}\right)$ | $O\left(\sum_k \frac{1}{\Delta_k^2}\log\frac{K}{\delta}\right)$ |
| Weak Quantum | $\Omega\left(\sum_k \frac{1}{\Delta_k}\log\frac{1}{\delta}\right)$ | $\tilde{O}\left(\sum_k \frac{1}{\Delta_k}\log\frac{1}{\delta}\right)$ |
| Strong Quantum | $\Omega\left(\sqrt{\sum_k \frac{1}{\Delta_k^2}}\right)$ | $\tilde{O}\left(\sqrt{\sum_k \frac{1}{\Delta_k^2}}\log\frac{1}{\delta}\right)$ |
| $m$-Constrained Quantum | $\Omega\left(\sum_{\mathcal{S}\in\mathfrak{B}}\sqrt{\sum_{k\in\mathcal{S}}\frac{1}{\Delta_k^2}}\right)$ | $\tilde{O}\left(\sum_{\mathcal{S}\in\mathfrak{B}}\sqrt{\sum_{k\in\mathcal{S}}\frac{1}{\Delta_k^2}}\right)$ |